B.Sc. 3rd Semester (Honours) Examination, 2022 (CBCS)

Subject : Physics

Course : CC-V

Full Marks: 40

Time: 2 Hours

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

2×5=10

1. Answer *any five* questions from the following:

(a) If 2π be the period of $\sin x$ then what will be the period of (i) $\sin 2\pi x$ and (ii) $\sin(4t-1)$?

(b) Write Parseval's identity and explain the terms therein.

(c) Normalize $P_2(x)$ in the interval $(-1, 1)$.

(d) Write down Hermite differential equation and check its singularity at $x = 0$.

(e) Express the integral $\int_0^1 \sqrt[3]{lnx}dx$ as a Γ function.

(f) Show that $\beta(p+1, q) = \frac{p}{p+q}\beta(p, q)$.

(g) Prove that error function is an odd function of x.

(h) Write a short note on different types of systematic errors.

5×2=10

Answer *any two* questions from the following.

2. Expand

$f(x) = \sin x$ when $0 \leq x < \pi$

$= 0$ when $\pi \leq x \leq 2\pi$

Comment on the result. 4+1

3. Prove the following: 2+3

(a) $(2n+1)xP_n(x) = nP_{n-1}(x) + (n+1)P_{n+1}(x)$

(b) $(2n+1)P_n(x) = P'_{n+1}(x) - P'_{n-1}(x)$

4. Prove that $erfc(x) \cong \frac{e^{-x^2}}{x\sqrt{\pi}}\left[1 - \frac{1}{2x^2} + \frac{1.3}{(2x^2)^2} - \frac{1.3.5}{(2x^2)^3} + \cdots\right]$.

5. The displacement y of a viscously damped string is given by $\frac{\partial^2 y}{\partial t^2} = c^2\frac{\partial^2 y}{\partial x^2} - 2k\frac{\partial y}{\partial t}$. Find the general solution of the above equation by the method of separation of variables. (Consider small damping.)

Answer *any two* questions from the following: 10×2=20

6. (a) Find out the solution of the following differential equation:

$$x\frac{d^2y}{dx^2} + a\frac{dy}{dx} + k^2xy = 0$$

(b) What do you understand by propagation of errors? Measured value of length, L of a simple pendulum is 20·0 cm known to 1 mm accuracy and time for 100 oscillations of the pendulum is found to be 90s using a wrist watch of 1s resolution. What will be the accuracy in the determination of g? 6+1+3

7. (a) Prove that $\int_0^1 \frac{dx}{\sqrt{1-x^n}} = \frac{\sqrt{\pi}}{n}\frac{\Gamma\left(\frac{1}{n}\right)}{\Gamma\left(\frac{n+2}{2n}\right)}$.

(b) Prove that $\int_0^{\pi/2} \sqrt{\tan\theta}\, d\theta = \frac{1}{2}\beta\left(\frac{3}{4}, \frac{1}{4}\right)$.

(c) Prove that $\frac{\beta\left(\frac{p+1}{2}, \frac{1}{2}\right)}{\beta\left(\frac{p+1}{2}, \frac{p+1}{2}\right)} = 2^p$. 4+2+4

8. (a) Plot the given periodic function $f(x) = \begin{cases} -1, & -\pi < x < \frac{\pi}{2} \\ 1, & \frac{\pi}{2} < x < \pi \end{cases}$.

(b) Express $f(x) = x$ as a Fourier series in the interval, $-\pi < x < \pi$.

(c) Solve the equation by power series method: $4xy'' + 2y' + y = 0$ 1+3+6

9. (a) Prove that $\mathfrak{J}_{-p}(x) = (-1)^p \mathfrak{J}_p(x)$.

(b) At tightly stretched string with fixed end points at $x = 0$ and $x = l$ is initially in a position given by $y = y_0 \sin^3(\pi x/l)$. If it is released from rest from this position, find the displacement $y(x, t)$. 2+8
